CAM6 Deep Cumulus Parameterization

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CAM6 Deep Cumulus Parameterization: Structure of Problem

References:
Arakawa and Shubert (1974, J. Atmos. Sci.)
Zhang and McFarlane (1995, Atmosphere-Ocean)

\[
\bar{\rho} \frac{\partial \bar{s}}{\partial t} + \nabla \cdot (\bar{\rho} \bar{v} \bar{s}) + \frac{\partial}{\partial z} (\bar{\rho} \bar{w} \bar{s}) = \bar{Q} \text{(Phase Changes)} + \bar{Q}_R
\]

\[s = c_p T + g z, \text{ dry static energy}\]

overbar – grid mean
Structure of Problem

1 = a_u + a_d + a_e

M_u = \rho_u w_u a_u
M_d = \rho_d w_d a_d

M_c = M_u + M_d

\bar{\rho} \frac{\partial \bar{s}}{\partial t} + \nabla \cdot (\bar{\rho} \bar{v} \bar{s}) = - \frac{\partial}{\partial z} (M_u S_u + M_d S_d + \bar{\rho} a_e w_e \bar{s})

+ \bar{Q} (Phase Changes) + \bar{Q}_R
Structure of Problem

\[ \rho \bar{w} = \rho \alpha_e \bar{w}_e + M_c \]

\[ \bar{\rho} \frac{\partial \bar{s}}{\partial t} + \nabla \cdot (\bar{\rho} \bar{vS}) = -\frac{\partial}{\partial z} \left[ M_u S_u + M_d S_d + \bar{s}(\rho \bar{w} - M_c) \right] + \bar{Q} \text{(Phase Changes)} + \bar{Q}_R \]
Structure of Problem

\[ \overline{\rho \bar{w}} = \bar{\rho} a_e w_e + M_c \]

\[
\bar{\rho} \frac{\partial \bar{s}}{\partial t} + \nabla \cdot (\overline{\rho \bar{v} s}) = - \frac{\partial}{\partial z} \left[ M_u S_u + M_d S_d + \bar{s} (\overline{\rho \bar{w}} - M_c) \right] + \bar{Q} (\text{Phase Changes}) + \bar{Q}_R
\]

\[ \bar{\rho} \frac{\partial \bar{s}}{\partial t} + \nabla \cdot (\overline{\bar{V} \ s \bar{\rho}}) + \frac{\partial}{\partial z} (\overline{\rho w s}) = - \frac{\partial}{\partial z} (M_u S_u + M_d S_d - \bar{s} M_c) + \bar{Q} (\text{Phase Changes}) + \bar{Q}_R \]
Structure of Problem

\[ C_p \left( \frac{\partial T}{\partial t} \right)_{Cu} = -\frac{1}{\rho} \frac{\partial}{\partial z} \left( M_u S_u + M_d S_d - \bar{s}M_c \right) \]
\[ + Q(\text{Convective Phase Changes}) \]
\[ + Q_R(\text{Convective Clouds}) \]

Also,

\[ \left( \frac{\partial q}{\partial t} \right)_{Cu} = -\frac{1}{\rho} \frac{\partial}{\partial z} \left( M_u q_u + M_d q_d - \bar{q}M_c \right) + ... \]

Similarly, for tracers, horizontal momentum, microphysics.

Deep cumulus parameterization provides:
\[ M_u, M_d, M_u S_u, M_d S_d, M_u q_u, M_u q_d, \]
convective phase changes, ...

Updrafts, $M_u$


Updrafts are an ensemble of entraining plumes:

$$
\frac{1}{m} \frac{dm}{dz} = \lambda
$$
Updrafts, $M_u$


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$$\frac{1}{m} \frac{dm}{dz} = \lambda$$

Defining Assumption: An alternate is GFDL AM4 (Zhao et al., 2018a,b, JAMES) which uses a variant of Bretherton et al.’s (2004, Mon. Wea. Rev.) buoyancy-sorting, entrainment-detainment plume.
Updrafts, $M_u$


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Updrafts, $M_u$


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Deep plumes extend above minimum in grid-mean saturated moist static energy $\tilde{h}^* = c_p \bar{T} + g z + L q_{sat}(\bar{T})$

\[ m(z) = m_b(\lambda) e^{\lambda(z-z_b)} \]

Assume \( m_b(\lambda) \) independent of \( \lambda \) ->

Same base mass flux for all ensemble members
\[ m(z) = m_b(\lambda) e^{\lambda(z-z_b)} \]

Assume \( m_b(\lambda) \) independent of \( \lambda \) ->
Same base mass flux for all ensemble members

Defining Assumption: An alternate is GFDL AM3, where the base mass fluxes vary strongly among ensemble members, constrained by observations of updraft vertical velocities (Donner, 1993, *J. Atmos. Sci.*).
Updrafts

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Assume \( m_b(\lambda) \) independent of \( \lambda \) ->
Same base mass flux for all ensemble members
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Assume \( m_b(\lambda) \) independent of \( \lambda \) ->
Same base mass flux for all ensemble members

\( \lambda_0 \) : largest entrainment rate, for plume with zero buoyancy at \( \bar{h}^* \) minimum

\( \lambda_D(z) \) : entrainment rate for plume with zero buoyancy at \( z \)

\( \lambda_D(z_0) = \lambda_0 \)
If all plumes have same base mass flux, total ensemble mass flux at $z$ is

$$M_u(z) = M_b \int_0^{\lambda_D} \frac{1}{\lambda_0} e^{\lambda(z-z_b)} d\lambda$$

$$= \frac{M_b}{\lambda_0(z-z_b)} \left[ e^{\lambda_D(z-z_b)} - 1 \right]$$
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= \frac{M_b}{\lambda_0(z-z_b)} \left[ e^{\lambda_D(z-z_b)} - 1 \right]
\]

Solve for \( \lambda_D \):

\[
h_b - h^* = \lambda_D \int_{z_b}^{z} [h_b - h(z')] e^{\lambda_b(z'-z)} dz'
\]
If all plumes have the same base mass flux, the total ensemble mass flux at \( z \) is

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\( M_b \) must be determined by closure.
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\( M_b \) must be determined by closure

\( M_b, M_u, \lambda_D \) and plume equations can be used to determine updraft — related parameterization outputs.
Ensemble Downdraft Mass Flux, $M_d$

Proportional to Cloud-Base Mass Flux
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\[ M_d(z) = -\frac{\alpha M_b}{\lambda_m(z_D - z)} \left[ e^{\lambda_m(z_D - z)} - 1 \right] \]
Proportional to Cloud-Base Mass Flux

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- \( \lambda_m \), maximum downdraft entrainment rate
Proportional to Cloud-Base Mass Flux

\[ M_d(z) = - \frac{\alpha M_b}{\lambda_m(z_D - z)} \left( e^{\lambda_m(z_D - z)} - 1 \right) \]

- \( \lambda_m \), maximum downdraft entrainment rate
- start at lower of updraft detrainment layer or minimum \( \bar{h} \)
Downdrafts

Proportional to Cloud-Base Mass Flux

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- \( \alpha = \mu \left( \frac{P}{E_d+P} \right) \), where \( P \) is precipitation and \( E_d \)
  is evaporation req’d to maintain saturation in downdraft
Proportional to Cloud-Base Mass Flux

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- \( \alpha = \mu \left( \frac{P}{E_d + P} \right) \), where \( P \) is precipitation and \( E_d \) is evaporation req’d to maintain saturation in downdraft
- \( \mu \) specified (\( \sim 0.2 \))
Cloud-Base Mass Flux, $M_b$

Assume convection consumes Convective Available Potential Energy (CAPE). CAPE is energy released by an undilute parcel moving from its Level of Free Convection (LFC) to its Level of Zero Buoyancy (LZB).
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\[
CAPE = g \int_{z(LFC)}^{z(LZB)} \frac{\theta_{vp} - \bar{\theta}_v}{\bar{\theta}_v} \, dz
\]

\(\theta_v\) is virtual potential temperature subscript in \(\theta_{vp}\) indicates parcel

\[
\left. \frac{\partial CAPE}{\partial t} \right|_{Cu} = - M_b F
\]

cape.png from NWS JetStream Max: Severe Weather: https://www.weather.gov/jetstream/skewt_severe_max
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CAPE = g \int_{z(LFC)}^{z(LZB)} \left( \frac{\theta_{vp} - \bar{\theta}_v}{\bar{\theta}_v} \right) dz
\]

\[\theta_v\] is virtual potential temperature subscript in \(\theta_{vp}\) indicates parcel \(M_b F\)

\[
\left( \frac{\partial CAPE}{\partial t} \right)_{Cu} = - M_b F
\]

obtain F by differentiating CAPE integral, making use of \(\left( \frac{\partial T}{\partial t} \right)_{Cu}\) and \(\left( \frac{\partial q}{\partial t} \right)_{Cu}\)
Closure Assumption: Convection consumes CAPE generated by large-scale processes, e.g., radiative cooling and advection ("quasi-equilibrium"). Approximate as $\frac{CAPE}{\tau}$, where $\tau$ is an imposed relaxation time scale.

So, $M_b = - \frac{CAPE}{\tau F}$
Bulk Plume Microphysics and Phase Changes

\[ \frac{\partial h_u}{\partial z} = \lambda (\bar{h} - h_u) \quad z_b \leq z \leq z_D \]
Bulk Plume Microphysics and Phase Changes

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\frac{\partial (M_u S_u)}{\partial z} = (E_u - D_u) \bar{s} + L C_u
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\frac{\partial (M_u q_u)}{\partial z} = E_u \bar{q} - D_u q_u - C_u
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\[ \frac{\partial (M_uq_u)}{\partial z} = E_u \bar{q} - D_uq_u - C_u \]

\[ \frac{\partial (M_u l)}{\partial z} = -D_u l + C_u - P \]
Bulk Plume Microphysics and Phase Changes

\[ \frac{\partial h_u}{\partial z} = \lambda (\bar{h} - h_u) \quad \text{for} \quad z_b \leq z \leq z_D \]

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- \( l \): ensemble mean liquid water
Bulk Plume Microphysics and Phase Changes

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- Terms proportional to \( M_b \)
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- \(l\): ensemble mean liquid water
- Terms proportional to \(M_b\)
- \(P\), precipitation, \(P = C_0 M_b l\)
- \(C_0\), autoconversion of cloud condensate to precipitation
CAM Extensions / Modifications to Zhang – McFarlane Parameterization

• *Deep Convective Horizontal Momentum Transport* (Richter and Rasch, 2008, *J Clim*)

\[
\frac{\partial \bar{V}}{\partial t}_{cu} = - \frac{1}{\rho} \frac{\partial}{\partial z} \left( M_u \bar{V}_u + M_d \bar{V}_d - M_c \bar{V} \right)
\]
CAM Extensions / Modifications to Zhang – McFarlane Parameterization

- Deep Convective Horizontal Momentum Transport (Richter and Rasch, 2008, J Clim)

\[
\frac{\partial \bar{V}}{\partial t} \bigg|_{Cu} = - \frac{1}{\rho} \frac{\partial}{\partial z} (M_u V_u + M_d V_d - M_c V) \\
- \frac{1}{\rho} \frac{\partial}{\partial z} (M_u V_u) = E_u V - D_u V_u + \vec{P}_G^u
\]
CAM Extensions / Modifications to Zhang – McFarlane Parameterization

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  (Richter and Rasch, 2008, *J Clim*)

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- \frac{1}{\rho} \frac{\partial}{\partial z} (M_u V_u) = E_u V - D_u V_u + \vec{P}_G^u
\]

\[
- \frac{1}{\rho} \frac{\partial}{\partial z} (M_d V_d) = E_d V + \vec{P}_G^d
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CAM Extensions / Modifications to Zhang – McFarlane Parameterization

- **Deep Convective Horizontal Momentum Transport** (Richter and Rasch, 2008, *J Clim*)

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\[- \frac{1}{\rho} \frac{\partial}{\partial z} (M_d V_d) = E_d V + \vec{P}_G^d
\]

with pressure-gradient forces:

\[
\vec{P}_G^u = - C_u M_u \frac{\partial V}{\partial z}
\]

\[
\vec{P}_G^d = - C_d M_d \frac{\partial V}{\partial z}
\]

\(C_u, C_d\) tunable parameters
• **Deep Convective Tracer Transport**

\[
\frac{\partial}{\partial p} (M_x q_x) = E_x \bar{q} - D_x q_x
\]

\[x \in (u, d)\]
• **Deep Convective Tracer Transport**

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\frac{\partial}{\partial p} (M_x q_x) = E_x \bar{q} - D_x q_x
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• **Evaporation of Convective Precipitation**
(Sundqvist, 1988, *Physically Based Modelling*)

proportional to \((1 - \text{RH}(z)) \ [P(z)]^{1/2}\)
• **Deep Convective Tracer Transport**

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proportional to \((1-\text{RH}(z)) \left[\text{P}(z)\right]^{1/2}\)

• **CAM6 tuned to increase sensitivity to convective inhibition**